Adaptive Cruise Control Design Using Reach Control

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***Abstract*— We investigate a correct-by-construction synthesis of piecewise affine feedback controllers designed to satisfy the strict safety specifications set forth by the adaptive cruise control (ACC) problem. Our design methodology is based on the formulation of the ACC problem as a reach control problem on a polytope in a 2D state space. The boundaries of this polytope, expressed as linear constraints on the states, arise from the headway and velocity safety requirements imposed by the ACC problem statement. We propose a model for the ACC problem, develop a controller that satisfies the ACC requirements, and produce simulations for the closed-loop system.**

I. Introduction

Adaptive cruise control is a mechanism that seeks to control a vehicle's speed while maintaining a safe distance from a preceding lead vehicle, using sensors solely within the actuated vehicle. In this paper we focus on an approach that casts the problem as one of control synthesis to achieve a Linear temporal logic (LTL) specification [4], [5], [11], [12]. LTL-based ACC designs were presented in [6], [7], [8]. The main step is to construct a finite state *transition system* that accurately captures the continuous time dynamics of the control system. The transition system may be constructed using numerical tools that discretize time, the state space, and/or the input space. The disadvantage of such numerical algorithms is that they are approximate and may overlook simpler solutions.

In this paper, we adopt the vehicle model and the LTL specification from [7]. In contrast to [7], we employ a (hand-crafted) partition of the state space consisting of a small number of polytopic regions (simplices). This hand-crafted partition may seem adhoc, but for the ACC problem it results in a particularly simple design. The (high-level) LTL synthesis is then straightforward, being informed by observations about the 2D dynamics of the control system. Finally, low-level feedback controllers that implement the high-level specification are designed on each polytopic region using reach control theory [1], [2], [10].

The basic idea of reach control theory is depicted in Figure 1 for a 2D state space. The polytopic state space is triangulated into simplices, and on each simplex an affine controller is devised that forces trajectories starting in the simplex to move to the next one in the sequence (the sequence is usually determined by a high-level dynamic programming algorithm) so that, overall, an LTL specification is met. Further details are described in Section III.

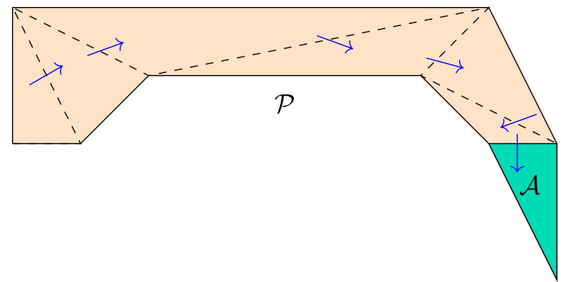

Fig. 1. The reach control approach for solving LTL control problems. The system state trajectories are required to enter the area $\mathcal{A}$ without previously leaving the state space $\mathcal{P}$. The state space is triangulated into simplices, and a controller is found on each simplex so that it drives the system states to enter a subsequent simplex.

The contribution of this paper is to reconsider the ACC problem as an LTL control synthesis problem with a solution based on reach control theory. The benefit is to obtain an elegant design with a computational complexity that is negligible, with strict guarantees on safety, offering the robustness of feedback control, and without ever resorting to numerical methods or approximation. In summary, the advantages of our approach are: **simplicity**: the partition (a triangulation) consists of eight simplices, rather than the hundreds or even thousands of polytopic regions that would be generated by a numerical algorithm; **safety**: guarantees are built into the design using reach control theory; **computation**: the calculation of reach controllers for any speed of the lead vehicle is computationally trivial.

The downside of our solution is that we make no claims on solving the full practical problem. We offer a concrete solution based on a theoretical design.

II. Model and Problem Statement

We adopt the model of a one- and two-vehicle system in [7]. The actuated vehicle is modeled as a point mass m moving along a straight line at some speed v. The equations of motion are given by

$$m\dot{v} = F_u - F_f, \tag{1}$$

where F_u is the net braking action and engine torque exerted on the vehicle, and F_f is the net friction force. The friction force is modeled as

$$F_f = f_0 + f_1 v + f_2 v^2.$$

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We assume $F_{br} \le F_u \le F_{ac}$, where F_{br} and F_{ac} are maximal braking and acceleration forces, respectively.

In addition to the actuated vehicle dynamics, we must consider the headway h between the lead vehicle and the actuated vehicle. We assume the lead vehicle is moving at a known speed which is a function of time; that is, $v_L : [0, +\infty) \to [v_{min}, v_{max}]$, where v_{min} and v_{max} are some reasonable constant minimal and maximal vehicle speeds. These parameters depend on the type of highway, weather conditions, and other characteristics.

The dynamics of the headway in the two-vehicle system can be described by:

$$\dot{h} = v_L - v. \tag{2}$$

By combining (1) and (2) we obtain the following two-vehicle dynamics:

$$\begin{pmatrix} \dot{v} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} -\frac{f_1}{m} & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} v \\ h \end{pmatrix} + \begin{pmatrix} \frac{1}{m} \\ 0 \end{pmatrix} F_u + \begin{pmatrix} \frac{-f_0}{m} \\ v_L \end{pmatrix} + \begin{pmatrix} \frac{-f_2}{m} v^2 \\ 0 \end{pmatrix}. \tag{3}$$

The value of f_2/m, given in Section VI-A, is extremely small in practice. Thus, we approximate the friction $F_f = f_0 + f_1 v + f_2 v^2$ by its linearization around $v = v_0 := (v_{min} + v_{max})/2$. This yields the following system

$$\begin{pmatrix} \dot{v} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} -\frac{f_1}{m} - 2\frac{f_2}{m} v_0 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} v \\ h \end{pmatrix} + \begin{pmatrix} \frac{1}{m} \\ 0 \end{pmatrix} F_u + \begin{pmatrix} \frac{-f_0}{m} + \frac{f_2}{m} v_0^2 \\ v_L \end{pmatrix}. \tag{4}$$

If v_L is constant, then system (4) is an affine control system with control input F_u. Thus, it is amenable to a reach control approach [2], [10]. The simulation results presented in Section VI-A show that there is no substantial difference between models (3) and (4); see also [7]. We proceed with our methodology based on system (4).

A. Problem Statement

Define the *time headway* $\tau = h/v$ which is the time required for a moving vehicle with velocity v to reach a stationary object at distance h from the vehicle. We summarize the ACC specifications from [3]:

1) ACC operates in two modes: *no lead car mode* and *lead car mode*.
2) In no lead car mode, a preset desired speed v_{des} eventually needs to be reached and maintained.
3) In lead car mode, a desired lower bound on safe time headway τ_{des} to the lead vehicle and an upper bound on a desired velocity v_{des} eventually need to be reached and maintained.
4) The time headway τ must be greater than $\tau_{min} = 1$s at all times.
5) Independently of the mode, the input F_u belongs to the admissible control set $\mathcal{U} := \{F_u \mid F_{br} \le F_u \le F_{ac}\}$.

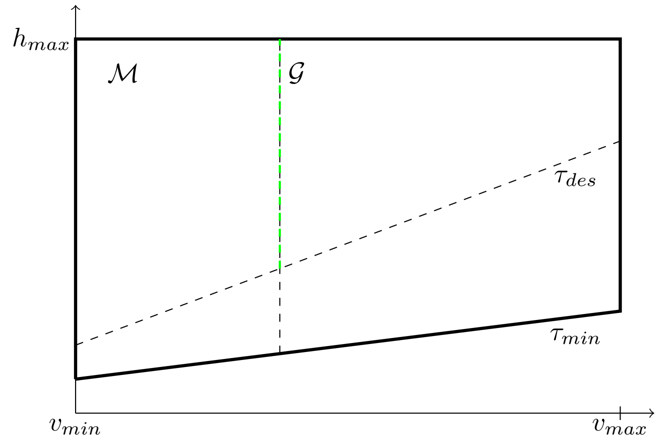

Fig. 2. The state space $\mathcal{M}$ is a polytope bounded by lines $v = v_{min}$, $v = v_{max}$, $h/v = \tau_{min}$ and $h = h_{max}$. The desired time headway τ_{des} and lead vehicle speed v_L are marked by dashed lines. The goal set $\mathcal{G}$ is marked in green.

The desired behaviour of the vehicle in the no lead car mode is trivially achievable using a variety of classical cruise control techniques, so it will not be discussing further. We impose a soft constraint $h \le h_{max}$, representing that it is not desirable for a car to fall behind the lead vehicle by more than h_{max}. If this does occur, the ACC switches from the lead car mode to the no lead car mode. Specification 4) together with the specification $h \le h_{max}$ determine the constrained polytopic state space, which is given by

$$\mathcal{M} := \{(v, h) \mid v_{min} \le v \le v_{max}, \tau_{min} v \le h \le h_{max}\}.$$

Finally, specification 3) defines the desired goal set $\hat{\mathcal{G}} = \{(v, h) \in \mathcal{M} \mid v \le v_{des}, h \ge \tau_{des} v\}$. If $v_{des} < v_L$, the actuated vehicle, upon reaching the goal set, will falling behind the lead vehicle and leave $\mathcal{M}$. Thus, we do not discuss this case further, and we assume $v_{des} \ge v_L$. On the other hand, if $v_{des} > v_L$, as v approaches the desired speed, the headway will be decreasing and eventually violate the specification. Hence, we impose that $v_{des} = v_L$. Then the goal set is

$$\mathcal{G} := \{(v, h) \in \mathcal{M} \mid v = v_L, h \ge \tau_{des} v\} \subset \hat{\mathcal{G}}.$$

Note that in the case that v_L is variable, $\mathcal{G}$ varies with time, whereas $\mathcal{M}$ and $\mathcal{U}$ remain the same. Figure 2 shows the state space $\mathcal{M}$ and the goal space $\mathcal{G}$ for some value of v_L. For $x_0 \in \mathcal{M}$ and a control function F_u, let $\phi_{F_u}(\cdot, x_0)$ be the trajectory of the system (4) with $\phi_{F_u}(0, x_0) = x_0$. We formulate the ACC problem as follows:

Problem 1: Find a control function F_u taking values in $\mathcal{U}$ such that, for all $x_0 \in \mathcal{M}$, the following holds:

(i) $\phi_{F_u}(t, x_0) \in \mathcal{M}$ for all $t > 0$,
(ii) $\lim_{t \to +\infty} \phi_{F_u}(t, x_0) \in \mathcal{G}$.

III. Methodology of Reach Control

The overarching goal of reach control theory (see, e.g., [2], [10]) is to address complex control specifications such as LTL specifications on a state space constrained by linear

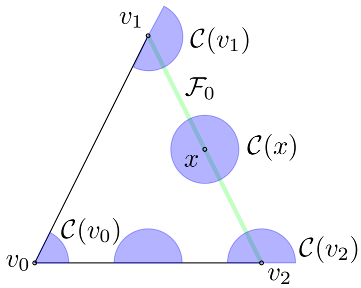

Fig. 3. Simplex $\mathcal{S} \subset \mathbb{R}^2$ with vertices v_0, v_1, v_2 and facets $\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2$. The exit facet $\mathcal{F}_0$ is marked in green. Cones $\mathcal{C}(x)$ are calculated at select points $x \in \mathcal{S}$ and drawn attached to those points.

inequalities. The state space is first partitioned into simplices or convex polytopes. Generally, the desire is for each simplex in the partition to either serve as a transition to another simplex or to contain an equilibrium point to which the trajectories should converge. In our exposition, we concentrate on the case where trajectories inside a simplex $\mathcal{S}$ need to exit $\mathcal{S}$ through a predetermined facet $\mathcal{F}_0$, thereby entering a neighbouring simplex $\mathcal{S}'$.

Consider an n-dimensional simplex $\mathcal{S} \subset \mathbb{R}^n$ with vertices $v_0, \ldots, v_n$. Let its facets be denoted by $\mathcal{F}_0, \ldots, \mathcal{F}_n$, where each facet is indexed by the vertex it does not contain. Let h_j be the unit normal vector to each facet $\mathcal{F}_j$ pointing outside of the simplex. Define the index set $I := \{1, \ldots, n\}$ and for each $x \in \mathcal{S}$ define $I(x)$ to be the minimal index set among $\{0, \ldots, n\}$ such that $x \in \text{co}\{v_i \mid i \in I(x)\}$, where co denotes the convex hull.

We study an affine control system

$$\dot{x} = Ax + Bu + a \tag{5}$$

on $\mathcal{S}$, where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $a \in \mathbb{R}^n$. Let $\mathcal{B} = \text{Im}(B)$, the image of B, and let $\Phi_u(\cdot, x_0)$ denote the trajectory of (5) starting at x_0 under input u.

Remark 2: The parameter $a \in \mathbb{R}^n$ in (5) is included for two reasons. First, it arises when linearizing a nonlinear system about an non-equilibrium point, a requirement to solve LTL problems on linear control systems [4]. Secondly, it is desirable to work with a class of systems closed under affine and piecewise affine feedbacks, since such feedbacks form the foundations of reach control theory [2], [10].

The main building block of reach control theory is the Reach Control Problem (RCP), to drive the trajectories from one simplex to a neighbouring simplex by forcing them to leave through an appropriate simplex facet.

Problem 3 (Reach Control Problem (RCP)):
Consider the system (5) defined on a simplex $\mathcal{S}$. Find a state feedback $u = u(x)$ such that for each $x_0 \in \mathcal{S}$ there exist $T \geq 0$ and $\varepsilon > 0$ such that

(i) $\Phi_u(t, x_0) \in \mathcal{S}$ for all $t \in [0, T]$,
(ii) $\Phi_u(T, x_0) \in \mathcal{F}_0$,
(iii) $\Phi_u(t, x_0) \notin \mathcal{S}$ for all $t \in (T, T + \varepsilon)$.

The RCP states that all trajectories in $\mathcal{S}$ must exit $\mathcal{S}$ through the *exit facet* $\mathcal{F}_0$ in finite time without first leaving $\mathcal{S}$ through any other *restricted* facet.

We now discuss a design procedure for solving the RCP for a given simplex using affine feedback. The first step is to choose control values $u_0, \ldots, u_n \in \mathbb{R}^m$ (one control for each vertex of the simplex) so that closed-loop trajectories do not cross the *restricted facets*. Mathematically, we require

$$Av_i + Bu_i + a \in \mathcal{C}(v_i)\,, \quad i = 0, \ldots, n\,, \tag{6}$$

where each $\mathcal{C}(v_i)$ is a closed, convex cone given by

$$\mathcal{C}(x) := \{y \in \mathbb{R}^n \mid h_j \cdot y \leq 0, j \in I \backslash I(x)\}\,. \tag{7}$$

Figure 3 illustrates these cones. Each cone characterizes the allowable vectors $Ax + Bu + a$ at each point $x \in \mathcal{S}$ so that closed-loop trajectories do not cross restricted facets. The conditions (6) are collectively called *invariance conditions*. Each invariance condition is a linear feasibility problem which can be solved via a linear program [2].

The second step is to compute an affine feedback $u(x) = Kx + g$ based on the chosen values $u_0, \ldots, u_n \in \mathbb{R}^m$; a formula is given in [2], [10]. Finally, the obtained affine feedback $u(x)$ solves the RCP on $\mathcal{S}$ if and only if $Ax + Bu(x) + a \neq 0$ for all $x \in \mathcal{S}$ [2], [10].

IV. Constant Speed Lead Vehicle

Returning to the ACC problem, we want to apply a reach control approach to guarantee trajectories starting in $\mathcal{M}$ reach the goal set $\mathcal{G}$. Consider the vertex $v^* = (v_{max}, \tau_{min} v_{max})$ at the lower right corner of $\mathcal{M}$. This is the point at which the actuated vehicle is moving at high speed and is close to the lead vehicle. If the lead vehicle is slow, the maximal braking effort may not be sufficient to keep the system state within $\mathcal{M}$. In the context of reach control theory, this means that the invariance condition (6) would not be solvable at v^* for any control $u \in [F_{br}, F_{ac}]$.

Hence, to guarantee safety we must to remove a part of $\mathcal{M}$ where it is impossible to satisfy the invariance conditions. A method for determining which parts of $\mathcal{M}$ need to be removed is discussed in [7]. For simplicity, we overapproximate the set that needs to be removed - we just remove a triangle $\mathcal{S}_0$ with vertices $(v_L, \tau_{min} v_L)$, $(v_{max}, \tau_{min} v_{max})$ and $(v_{max}, \tau_{min} v_L + b_{max})$, where

$$\begin{aligned} b_{max} &:= \frac{m(v_{max} - v_L)^2}{f_0' + f_1' \frac{v_{min} + v_{max}}{2} - F_{br}}, \\ f_0' &:= f_0 + f_1 v_0 + f_2 v_0^2, \\ f_1' &:= 2 f_2 v_0 + f_1. \end{aligned}$$

These values have been calculated so that the invariance conditions (6) are made feasible. We assume that $\tau_{min} v_L + b_{max} < h_{max}$ as it would otherwise be impossible to safely brake when the car is moving at the speed v_{max}.

Having removed $\mathcal{S}_0$, we now triangulate the remainder of the state space $\mathcal{M}$. The choice of triangulation is driven by our desired control strategy. We distinguish between four cases:

1° if the controlled vehicle is faster than the lead vehicle, and is following it a reasonably large distance, it can slow down to v_L while staying within the desired headway,

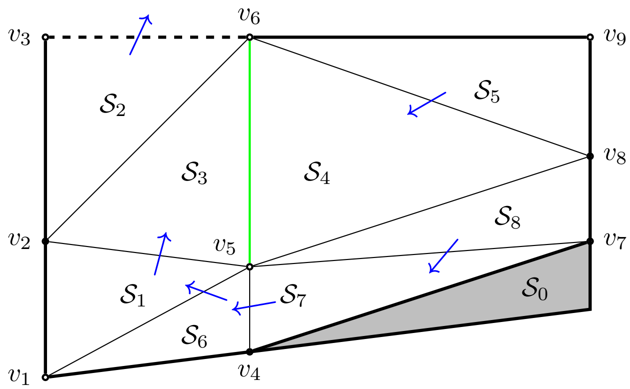

Fig. 4. Triangulation of the state space $\mathcal{M}$. Equilibrium points coinciding with the goal set $\mathcal{G}$ are marked in green. Exit facets of simplices are marked by blue arrows.

2° if the controlled vehicle is faster than the lead vehicle, but the distance between the two vehicles is small, the plan is to first slow down to below v_L, and then gradually increase the headway (see the next case),

3° if the controlled vehicle is slower than the lead vehicle, and the headway is small, it can increase its speed to v_L while also reaching the desired headway,

4° if the controlled vehicle is slower than the lead vehicle, and the distance between the two vehicles is large, it may not be possible to speed up to v_L fast enough without reaching the maximum headway h_{max} first. This is a suboptimal, but safe scenario discussed in Section II-A. In that case, the vehicle should just increase its speed.

We now define vertices $v_1, \ldots, v_9$ to generate a triangulation of the state space $\mathcal{M} \backslash \mathcal{S}_0$. As with b_{max}, we define

$$b_{min} := \frac{m(v_{min} - v_L)^2}{F_{ac} - f_0' - f_1' \frac{v_{min} - v_{max}}{2}} .$$

We also assume that $h_{max} - b_{min} > \tau_{min} v_{min}$, because otherwise, it would not be possible for a vehicle at $v = v_{min}$ to achieve the goal set for any given headway, thus eventually exiting $\mathcal{M}$. The coordinates of vertices are then given as follows: $v_1 = (v_{min}, \tau_{min} v_{min})$, $v_2 = (v_{min}, h_{max} - b_{min})$, $v_3 = (v_{min}, h_{max})$, $v_4 = (v_L, \tau_{min} v_L)$, $v_5 = (v_L, \tau_{des} v_L)$, $v_6 = (v_L, h_{max})$, $v_7 = (v_{max}, \tau_{min} v_L + b_{max})$, $v_8 = (v_{max}, \min\{\tau_{des} v_L + b_{max}, h_{max}\})$, and $v_9 = (v_{max}, h_{max})$. Observe that vertices v_1, v_3, v_4, v_7, and v_9 are vertices of the trimmed state space $\mathcal{M} \backslash \mathcal{S}_0$. Vertices v_5 and v_6 are vertices of the goal set $\mathcal{G}$. Vertices v_2 and v_8 have been chosen so that the segments $\overline{v_2 v_6}$ and $\overline{v_5 v_8}$ form a boundary between cases 1° and 2° and cases 3° and 4°, respectively. In the context of the RCP, this corresponds to ensuring that the invariance conditions (6) are satisfied on all simplices for the desired exit facets. Our triangulation of $\mathcal{M}$ is given in Figure 4. On each simplex we also denoted an exit facet that the system states are permitted to go through, or a set of equilibrium points that we want to reach, corresponding to our stated control strategy above.

Simplices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_5, \ldots, \mathcal{S}_8$ contain both an exit facet and an equilibrium point. In those cases, we wish to allow that a trajectory either converges to an equilibrium point or leaves the simplex through an exit facet.

Finally, it remains to design controls on each simplex $\mathcal{S}_i$, $i = 1, \ldots, 8$, to ensure the desired state behaviour. We propose these control values F_{ui} at vertices v_i:

$$\begin{aligned} F_{u1}, F_{u2}, F_{u3} &= F_{ac} , \\ F_{u4}, F_{u7}, F_{u8}, F_{u9} &= F_{br} , \\ F_{u5}, F_{u6} &= F_f' , \end{aligned}$$

where F_f' is the force which results in the vehicle speed not changing from v_L. A control law $u : \mathcal{M} \backslash \mathcal{S}_0 \to [F_{br}, F_{ac}]$ is then obtained by affinely extending the above control values to each triangle $\mathcal{S}_i$. This results in a continuous piecewise affine control law. It can be computationally verified that all of these input values indeed satisfy the desired invariance conditions. Direct calculations also show that $Ax + Bu(x) + a = 0$ if and only if $x \in \mathcal{G}$.

Finally, in order to guarantee that our proposed control law indeed satisfies the control objectives, let us discuss a theoretical detail. As noted above, the setup of simplices $\mathcal{S}_i$ is not exactly the same as the RCP setup in Section III. In Section III, simplex $\mathcal{S}$ had an exit facet and no desired equilibria. In our case, there are two different situations:

- The desired behaviour on each of the simplices $\mathcal{S}_i$, $i = 1, 2, 5, \ldots, 8$ requires the trajectory to either leave a simplex through an facet *or* converge to a single equilibrium $e \in \mathcal{G}$ which is on the boundary of $\mathcal{S}_i$. As mentioned above, it can be shown that the controller on $\mathcal{S}_i$ proposed above solves the invariance conditions (6). It was proved in [9] that, if a control input satisfies the invariance conditions, all trajectories that do leave a simplex indeed leave it through a designated facet. Thus, *if* a trajectory leaves a simplex $\mathcal{S}_i$, it will leave it through a desired exit facet.
  As mentioned above, the proposed controller also results in (5) on $\mathcal{S}_i$ containing a single equilibrium at e. Hence, because (5) on $\mathcal{S}_i$ is a two-dimensional affine system with a single equilibrium on the boundary of $\mathcal{S}_i$, if a trajectory does not leave $\mathcal{S}_i$, it necessarily converges to this equilibrium. The trajectory cannot diverge because it stays inside the simplex, and it cannot be periodic because that would require it to move around the equilibrium, which is not possible because e is on the boundary of $\mathcal{S}_i$.
- The desired behaviour on simplices $\mathcal{S}_i$, $i = 3, 4$, is to converge towards the facet $\mathcal{G}$ without leaving the simplex. As mentioned, we can show that the control law proposed above solves the invariance conditions on $\mathcal{S}_i$ (with $\mathcal{G}$ defined as the "exit facet"). Thus, by [9] the trajectories will either leave through $\mathcal{G}$ or not leave $\mathcal{S}_i$ at all. However, since $\mathcal{G}$ consists solely of equilibria of the system (5), it is impossible to leave through $\mathcal{G}$. Hence, all trajectories remain inside $\mathcal{S}_i$, and as a result, there is no chattering between $\mathcal{S}_3$ and $\mathcal{S}_4$.
  Moreover, since (5) on $\mathcal{S}_i$ is a two-dimensional affine system where an entire segment $\mathcal{G}$ is an equilibrium, it

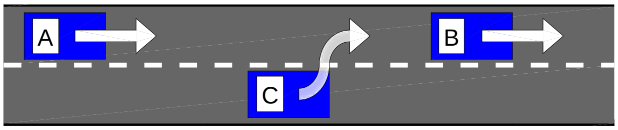

Fig. 5. Vehicle A under ACC is originally following lead vehicle B. Vehicle B speeds up, and vehicle C merges into the lane, becoming the new lead vehicle.

can easily be algebraically shown that the trajectories lie on straight lines. Since these trajectories need to remain inside $\mathcal{S}_i$ and there are no equilibria except on $\mathcal{G}$, the only option is that each trajectory converges to a point in $\mathcal{G}$.

V. Variable Speed Lead Vehicle

The proposed feedback control $u = u(x)$ on the state space $\mathcal{M}$ provides a correct-by-construction solution to the adaptive cruise control problem in the case when the lead vehicle speed v_L is constant. We now generalize this approach to a variable speed $v_L(t)$. We note that the triangulation defined in Figure 4 is with respect to v_L and is valid for any $v_{min} \leq v_L \leq v_{max}$. The control inputs F_{uj} defined at vertices v_j are also parametrized with respect to v_L. Thus, if $v_L = v_L(t)$ is a time-varying function, at every time $t \geq 0$ we can generate a triangulation $\mathbb{T}(t)$ of the state space $\mathcal{M}$, and using $\mathbb{T}(t)$ define a control function $u(x,t)$. The idea is that at every time instant t_0, the control $u(\cdot, t_0)$ will satisfy the invariance conditions imposed by a triangulation $\mathbb{T}(t_0)$ and will be driving system trajectories to converge to the goal set $\mathcal{G}(t_0)$. While the correctness of the controller $u(\cdot, t_0)$ is guaranteed for every fixed t_0, there is no guarantee that the time-varying controller $u(\cdot, \cdot)$ is still correct. Such a theory has not yet been developed in reach control; this paper is the first step towards developing it. The simulations presented in Section VI-A show that the above time-varying strategy indeed works.

VI. Simulation Results

In the following simulations, we use the following parameter values from [7]: $m = 1370$ kg, $f_0 = 3.8 \cdot 10^{-3} \cdot mg$ N, $f_1 = 2.6 \cdot 10^{-5} \cdot mg$ Ns/m, $f_2 = 0.4161$ Ns^2/m^2, $F_{br} = -0.3mg$ N, $F_{ac} = 0.2mg$ N. We additionally use $v_{min} = 15$ m/s, $v_{max} = 35$ m/s, $\tau_{min} = 1$s, $\tau_{des} = 2$s, $h_{max} = 300$m.

A. *Vehicle Merging in Front*

In this scenario, the vehicle we are controlling starts from a point in $\mathcal{G}$: $(v_0, h_0) = (v_L(0), (v_L(0)\tau_{des} + h_{max})/2) \in \mathcal{G}(0)$. The lead vehicle moves at a constant speed $v_L(t) = 30$, $0 \leq t \leq 10$. At time $T = 10$, a second vehicle merges from a neighbouring lane (see Figure 5). Hence, v_L now becomes the speed of the new lead vehicle, and is given by $v_L(t) = 25$, $t > 10$. Additionally, the new vehicle merging into the lane instantaneously reduced the time headway τ at time T to $\tau(T+) = (\tau_{min} + \tau_{des})/2$.

We note that in this case, both v_L and h are discontinuous, with breaks at time $T = 10$. The change in v_L requires

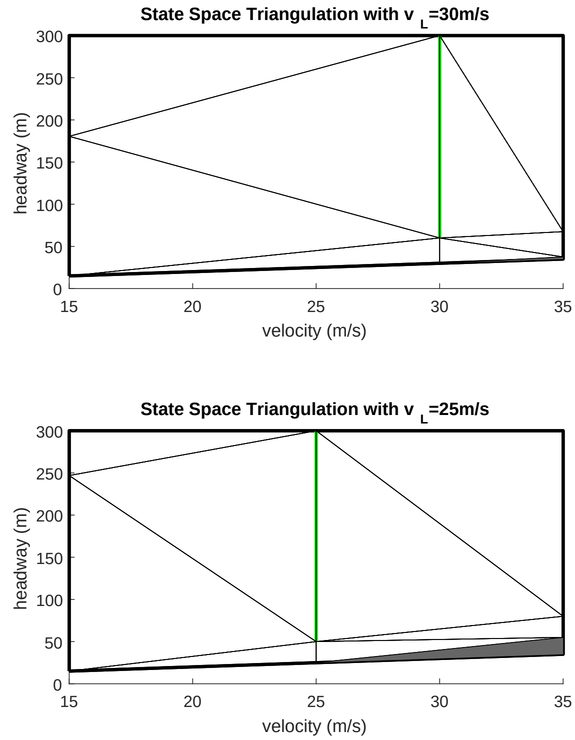

Fig. 6. Triangulations of the state space $\mathcal{M}$ as a function of the lead vehicle speed. The top figure shows $\mathbb{T}(t)$ for $0 \leq t \leq 10$. The bottom figure shows $\mathbb{T}(t)$, $t > 10$. The removed simplex $\mathcal{S}_0$ is shown in grey.

the triangulation $\mathbb{T}(t)$ to be recalculated at time T. Figure 6 shows the triangulations $\mathbb{T}(t)$ before and after a new vehicle merged into the lane.

In this simulation our car follows the nonlinear model (3). Unlike system (4), there are no guarantees that applying the feedback control $u(x,t)$ developed for the affine system (4) will result in a correct behaviour. However, the nonlinear system is well-approximated by an affine model. This is a result of the nonlinear factor $(v - v_0)^2 f_2/m$ being small in magnitude. Since the difference between trajectories produced by systems (3) and (4) are negligible, we only present the results of the simulations for system (3). We again simulate the behaviour of system (3) under such a scenario. The results are given in Figure 7.

We see that our controller performs well and tracks the speed of the lead vehicle while staying within the required safety envelope. Apart from minimal nonlinearity issues, this behavior was guaranteed by reach control theory.

We note that the action of the merging vehicle does not result in our vehicle being placed in an unsafe position. As such, our ACC strategy can still be used, just with new initial conditions $(v(T), v(T)\tau(T+)) \in \mathcal{M}$. Had the new lead vehicle merged into the lane in such a way that τ became smaller than the minimal safe headway τ_{min}, our controller would not function properly, as the state (v, h) would no longer be in $\mathcal{M}$ after time T. Intuitively, an emergency braking procedure should be invokved, but this is not covered by the ACC specification.

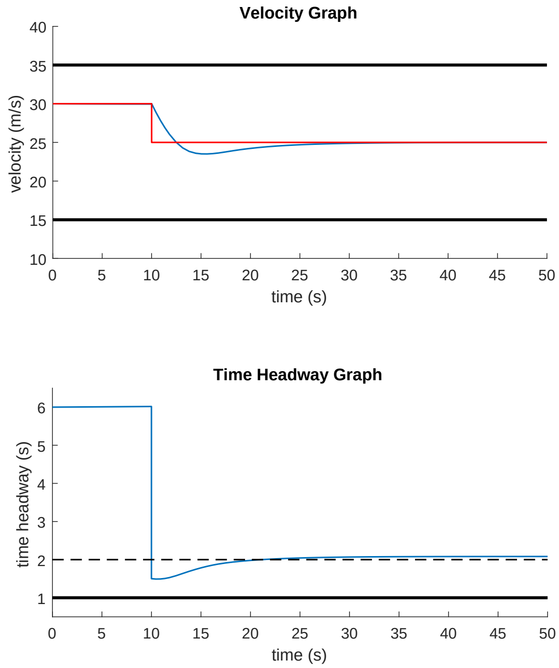

Fig. 7. Results of a simulation involving two lead vehicles with constant speed. The top graph shows the lead vehicle speed $v_L(t)$ over time (marked in red), as well as the speed of our vehicle $v(t)$ (marked in blue). The minimal and maximal velocity v_{min} and v_{max} are represented by solid black lines. The bottom graph shows the time headway $\tau(t)$, with the minimal allowed time headway τ_{min} represented by a solid black line and the minimal desired time headway τ_{des} represented by a dashed line.

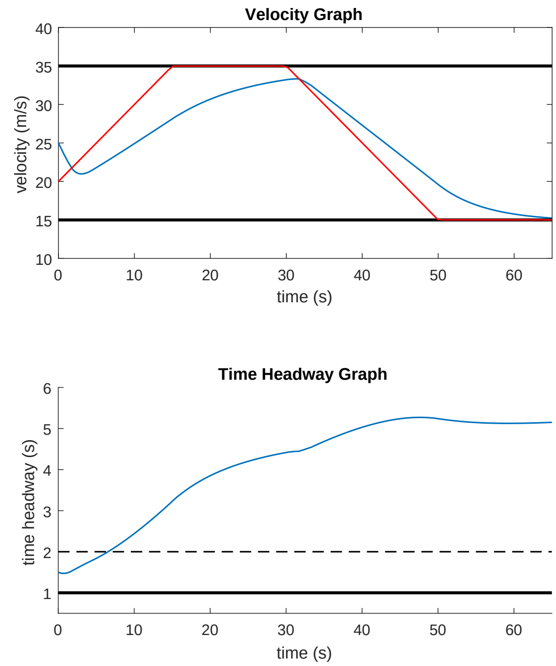

Fig. 8. Results of a simulation involving a single lead vehicle with nonconstant speed. The top graph shows the lead vehicle speed $v_L(t)$ over time and the speed of our vehicle $v(t)$. The bottom graph shows the time headway $\tau(t)$. As in Figure 7, minimal and maximal allowed velocity v_{min} and v_{max} are represented by solid black lines in the top graph, while in the bottom graph, the solid line represents the minimal allowed time headway τ_{min}, and the dashed line represents the minimal desired time headway τ_{des}.

B. Single Lead Vehicle With Nonconstant Speed

In this scenario, the vehicle we are controlling again follows the nonlinear model (3) and starts with the speed $v_0 = 25$, at $h_0 = 37.5$ meters behind the car in front. The lead car behaves according to the following velocity profile:

$$v_L(t) = \begin{cases} 20 + t, & 0 \leq t \leq 15, \\ v_{max}, & 15 \leq t \leq 30, \\ v_{max} - (t - 30), & 30 \leq t \leq 50, \\ v_{min}, & 50 \leq t \leq 65. \end{cases}$$

The results of the simulation are presented in Figure 8.

As we can see in Figure 8, $v(t)$ tracks $v_L(t)$ extremely well. In the segments when v_L is constant, $v(t)$ clearly converges towards v_L. Additionally, the vehicle speed remains between v_{min} and v_{max} at all times, and the time headway τ remains between τ_{min} and τ_{max}, eventually converging to τ_{des}. Unlike Section VI-A, there currently does not exist a theoretical guarantee for such behaviour, as the lead vehicle speed is constantly changing.

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