Incentive Design for Commercial Participation in Space Logistics Infrastructure Development and Deployment

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Abstract

The development and deployment of public space infrastructures, such as lunar habitat systems, in-situ resource utilization (ISRU) systems, and propellant depots, often require a large amount of initial investment that may not be affordable by a single stakeholder. It often requires global collaborations and federations of different governmental agencies and commercial entities. To analyze the interactions of stakeholders with different mission objectives and to stimulate commercial participation in future space infrastructure development and deployment, this paper proposes an incentive mechanism design framework based on game theory. An analytical incentive design method based on the Nash bargaining solution is established. We prove that this method can minimize the incentive required for commercial participation while maximizing both the social welfare (i.e., maximizing the total utility of all stakeholders) and the fairness (i.e., maximizing the minimum utility of each stakeholder). A case study on lunar habitat infrastructure deployment is implemented to demonstrate and evaluate the effectiveness of the proposed incentive design method. Results also show how the performances of ISRU system after deployment may impact the cooperation among stakeholders.

Keywords: Incentive Design, Space Logistics, Space Infrastructure, Game Theory

Nomenclature

$\mathcal{A}$	=	set of arcs
c	=	cost coefficient
d	=	mission demand
G	=	commodity transformation matrix
H	=	concurrency constraint matrix
i	=	node index
I_{sp}	=	specific impulse
j	=	node index
$\mathcal{J}$	=	space mission cost
k	=	player index
$\mathcal{K}$	=	set of players
$\mathcal{N}$	=	set of nodes
Q	=	baseline mission cost
r	=	disagreement point utility
t	=	time step index
$\mathcal{T}$	=	set of time steps
Δt	=	time of flight
u	=	mission utility
v	=	spacecraft index
$\mathcal{V}$	=	set of spacecraft
ΔV	=	change of velocity
W	=	set of time windows
x	=	commodity variable
α	=	participation coefficient
θ	=	incentive coefficient

1. Introduction

As the rocket launch cost decreases and technology develops, more government and commercial entities exhibit their interest in participating in future large-scale space exploration with their own mission objective preferences and technology advantages. Different from previous space exploration eras when logistically independent mission planning strategies were mainly implemented, for future deep space explorations beyond Earth orbits, space logistics infrastructures play an important role in reducing space mission cost leveraging mission interdependencies [1-6]. For example, in-situ resource utilization systems (i.e., oil fields in space) and propellant depots (i.e., gas stations in orbit) deployed in the early-stage of a space exploration campaign can support subsequent space missions by supplying propellant to transportation vehicles.

Multiple studies have been done leveraging space infrastructures to perform campaign-level space mission planning. These space logistics optimization frameworks were proposed to optimize space transportation scheduling and space infrastructure designs concurrently. They were developed based on heuristic methods [1], simulations [2], the graph theory [3], and commodity network flow models [4-6]. However, these studies assumed a single stakeholder that has a top-down control of all systems and resources during the space campaign.

On the other hand, space commercialization studies mainly focused on commercial opportunities in different industrial fields, such as the satellite industry [7,8] and the commercialization of the low-Earth orbit (LEO) [9]. These studies did not take into account specific space infrastructure development and deployment as part of the trade space.

The technology development and system deployment of these space public infrastructures are huge projects that rely on international collaborations and federations. Federated system (i.e., system-of-systems, SoS) is one of the important ways to stress the cooperation and competition among multiple actors. It has been widely used in the analysis of the collective decision-making process for federated satellite system [10], the government extended enterprises SoS [11], and the multi-actor space architecture commercialization [12]. However, even though successful federation can provide commercial players with higher rewards, they might still choose not to participate in the federation in fear of risk because of the unknowns of other players' decisions. Therefore, it requires the government/coordinator to offer incentives to pull the commercial players, who seek to maximize their own utilities, together. The problem is how to design an effective incentive mechanism to achieve this goal with minimum resources. To solve this problem, this paper proposes an incentive design framework based on game theory. The goal of the incentive mechanism design is not to stimulate cooperation among government and commercial entities but to attract more commercial participation and contributions to the space infrastructure development and deployment.

Mechanism design is a field in economics and game theory to design an incentive by implementing engineering approaches to achieve desired strategic settings in decision-making processes. It is also known as the reverse game theory because it considers the problem backward starting from the end of the game. It has been widely implemented in mobile phone participatory sensing [13-15], corporate entrepreneurship [16], and health risk assessment [17]. Specifically, a recent study by Kibiłda et al. [18] focusing on infrastructure deployment established an incentive design method for the mobile network market. Leveraging the current state-of-art in the incentive mechanism design field, we propose an incentive design framework for commercial participation in space infrastructure development and deployment. We establish the analytical incentive design method based on the Nash bargaining solution.

There are three contributions achieved in this paper. First, to the best of our knowledge, this study is the first incentive mechanism design research for future space infrastructure development and deployment. The proposed methods are beneficial to governmental space agencies and space mission coordinators to stimulate commercial participation in future space exploration. Second, our research extends the application of mechanism design and game theory to space commercialization through space logistics. It introduces a new perspective to interpret international federation with a diverse range of space activities and mission objectives from different space-faring countries. The proposed method is the first step towards building a macroeconomic framework for future space commercialization. Finally, the proposed incentive design framework is developed from the perspective of space logistics that takes into account space mission planning and space infrastructure design currently. It enables the commercial participation of governmental or commercial entities leveraging their previously deployed space infrastructures, such as space stations, lunar bases, ISRU systems, etc. It makes it possible to pull future "building blocks", which are multi-purpose space system-of-systems, in space exploration together. Our incentive design method is also particularly useful to evaluate the commercial potentials of space infrastructures and mission architectures.

The remainder of this paper is organized as follows. Section 2 first introduces the traditional space logistics optimization method for space infrastructure design and deployment. In Sec. 3, we perform a preliminary space mission cost analysis to establish a fundamental cost approximation model for space infrastructure deployment. In Sec. 4, we introduce the game theory model settings and definitions. In Sec. 5, we propose an incentive design method based on the Nash bargaining solution and analyze its properties. The performance and comparison of the proposed incentive design methods are then demonstrated in Sec. 6 through a lunar exploration campaign case study. Finally, Sec. 7 concludes the paper and discusses future works.

2. Traditional method: space logistics optimization

In this section, we introduce a traditional space logistics optimization method for space infrastructure development and deployment optimization. It assumes a single-player system with only one stakeholder that has a top-down control of all available technologies, architectures, and resources in the system. This optimization method formulates the space logistics optimization problem as a commodity network flow problem. In the network, nodes represent orbit or planet; arcs represent space flight trajectories; crew, propellant, instruments, spacecraft, and all other payloads are considered as commodities flowing along arcs, as shown in Fig. 1. A time dimension is introduced to take into account time steps for dynamic mission planning. There are mainly two types of arcs: 1) transportation arcs that connect different nodes at different time steps to represent space flights; 2) holdover arcs that connect the same node at different time steps to represent in-orbit or surface space mission operations after space infrastructure deployment.

Consider a network graph defined by a set of arcs $\mathcal{A} = \{\mathcal{V}, \mathcal{N}, \mathcal{T}\}$, which includes a spacecraft set $\mathcal{V}$ (index: v), a node set $\mathcal{N}$ (index: i and j), and a time step index set $\mathcal{T}$ (index: t). We define a commodity flow variable $\boldsymbol{x}_{vijt}$ as the decision variable for space mission planning, denoting the commodity flow from node i to node j at time step t using spacecraft v. It can be a continuous or discrete variable depending on the commodity type. For example, we need a discrete variable for the number of crew members or spacecraft; whereas we need a continuous variable for the mass of payload or propellant. To measure the space mission planning performance, we also define a cost coefficient parameter $\boldsymbol{c}_{vijt}$. The mission demands or supplies are determined through a demand vector $\boldsymbol{d}_{it}$, denoting the mission demand or supply in node i at time t. The mission demand is negative, and the mission supply is positive. If there are p types of commodities, $\boldsymbol{x}_{vijt}$, $\boldsymbol{c}_{vijt}$, and $\boldsymbol{d}_{it}$ are all $p \times 1$ vectors.

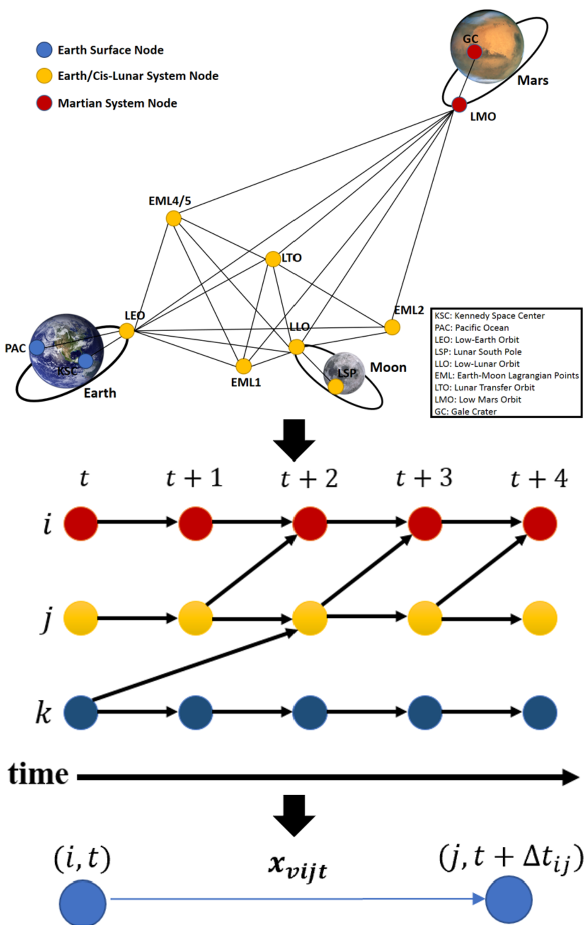

Fig. 1. Space logistics optimization network model

Besides the parameters defined above, we also need to define the following parameters for the formulation:

Δt_{ij} = time of flight.
G_{vij} = commodity transformation matrix.
H_{vij} = concurrency constraint matrix.
W_{ij} = mission time windows.

Based on the aforementioned notations, we can formulate a single-player network-based space logistics optimization model as follows:

Minimize:

$$\mathcal{J} = \sum_{(v,i,j,t)\in\mathcal{A}} \boldsymbol{c}_{vijt}{}^{T} \boldsymbol{x}_{vijt} \quad (1)$$

Subject to:

$$\sum_{(v,j):(v,i,j,t)\in\mathcal{A}} \boldsymbol{x}_{vijt} - \sum_{(v,j):(v,j,i,t)\in\mathcal{A}} G_{vji}\boldsymbol{x}_{vji(t-\Delta t_{ji})} \leq \boldsymbol{d}_{it} \quad \forall i \in \mathcal{N} \;\; \forall t \in \mathcal{T} \quad (2)$$

$$H_{vij}\boldsymbol{x}_{vijt} \leq \boldsymbol{0}_{l\times 1} \quad \forall (v,i,j,t) \in \mathcal{A} \quad (3)$$

$$\begin{cases} \boldsymbol{x}_{vijt} \geq \boldsymbol{0}_{p\times 1} & \text{if } t \in W_{kij} \\ \boldsymbol{x}_{vijt} = \boldsymbol{0}_{p\times 1} & \text{otherwise} \end{cases} \quad \forall (v,i,j,t) \in \mathcal{A} \quad (4)$$

$$\boldsymbol{x}_{vijt} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}_{vijt}, x_n \in \mathbb{Z}_+ \; or \; \mathbb{R}_+ \quad \forall n \in \{1, \dots, p\} \quad \forall (v,i,j,t) \in \mathcal{A}$$

In this formulation, Eq. (1) is the objective function. It minimizes the total mission cost of the space mission. Equation (2) is the commodity mass balance constraint. It guarantees that the commodity inflow is always larger or equal to the commodity outflow plus the mission demands. The second term $G_{kvji}\boldsymbol{x}_{kvji(t-\Delta t_{ji})}$ in this constraint represents commodity transformations during space flights or operations, including propellant burning, crew consumptions, and ISRU resource productions. For the detailed settings of the transformation matrix G_{vij}, please refer to Ref. [6]. Equation (3) is the concurrency constraint that defines the commodity flow upper bound typically determined by the spacecraft payload and propellant capacities. The index l is for the type of concurrency constraints considered. Equation (4) is the time bound. It defines the space flight time windows. Only when the time windows are open, the space transportation along an arc is permitted.

This section introduces a traditional space logistics optimization method for space infrastructure development and deployment in a single-player transportation system. This method can be extended to include multi-player decision-making process by adding another dimension for different players. However, there are two main limitations in implementing this traditional method directly to multi-player space infrastructure optimization. First, the game theory formulation, such as the bargaining problem, may introduce nonlinear terms in the formulation. Second, multi-player optimization introduces another dimension in the optimization. These two factors can significantly increase the size of the optimization problem. As a network-based commodity flow model, which is formulated as mixed-integer linear programming, this optimization problem is an NP-hard problem. The computation time of this optimization increases exponentially as the increase the problem size. As a result, implementing this traditional space logistics optimization method to the incentive design for multi-player space infrastructure development and deployment will lead to a scalability issue because of the curse of dimensionality. To solve this problem efficiently, in the following sections, we first extrapolate a mission cost approximation model based on this traditional method. Then, we propose an analytical game theory model for the incentive design in the multi-player decision-making environment.

3. Preliminary: space mission cost approximation

To build an analytical incentive design and optimization framework, we need to know the relationship between the space mission cost and the mass of space infrastructure to be deployed. In this section, we perform a sensitivity analysis through a lunar exploration campaign optimization to extrapolate the mathematical relationship between mission cost and the space infrastructure deployment demand.

First, we consider a cislunar transportation system, containing Earth, low-Earth orbit (LEO), Earth-Moon Lagrange point 1 (EML1), and the Moon. It is a four-node transportation network model, as shown in Fig. 2, where the discrete time-expanded network model uses one day as one time step. The space transportation Δv and time of flight (TOF) is also shown in Fig. 2.

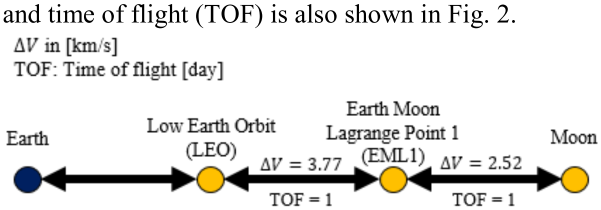

Fig. 2. Cislunar transportation network model

For the spacecraft (s/c) design, we assume all stakeholders to perform the space infrastructure deployment mission has the same spacecraft with RL-10 liquid hydrogen and liquid oxygen (LH2/LOX) rocket engines. Based on the spacecraft sizing of the ACES [19] spacecraft and the Centaur [20] spacecraft, we assume the transportation spacecraft considered has a dry mass of 6,000 kg and an inert mass fraction of 0.1. In the transportation system, we assume the stakeholder may also have deployed ISRU systems in advance to support the infrastructure deployment. The ISRU system is assumed as a water electrolysis ISRU, which generates oxygen and hydrogen from water. The baseline productivity of the ISRU is 5 kg water/yr/kg plant [19], which means 1 kg ISRU plant can electrolyze 5 kg water per year on the lunar surface. We also take into account the ISRU system maintenance, which requires maintenance spare resupply annually. The mass of maintenance spares required is equivalent to 5% of the ISRU system mass. The space mission operation parameters and assumptions are summarized in Table 1.

Table 1 Mission parameters and assumptions

Parameters	Assumed value
S/c propellant capacity, kg	54,000 [19, 20]
S/c structure mass, kg	6,000 [19, 20]
Propellant type	LH2/LOX
Propellant I_{sp}, s	420
Propellant O_2: H_2 ratio	5.5
Water ISRU productivity, kg H_2O/ yr/ kg plant	5 [19]
ISRU maintenance	5% plant mass

To analyze the space mission cost, we also need a space mission cost model. In this paper, we use the same mission cost model proposed by Chen et al. in a multi-actor space commercialization study [12]. It includes rocket launch cost to LEO, spacecraft manufacturing cost, space flight operation cost, and LH2/LOX propellant price on Earth. The cislunar transportation cost model is listed in Table 2.

Table 2 Cislunar tranportation cost model [12]

Parameter	Assumed value
Rocket Launch cost, /kg	\$3,500
S/c manufacturing, /#	\$148M
S/c operation, /flight	\$1M
LH2 price on Earth, /kg	\$5.94
LO2 price on Earth, /kg	\$0.09

According to the transportation network model, space operation assumptions, and the cost model discussed above, we perform numerical experiments to extrapolate the proper mathematical model of the mission cost approximation with respect to the mass of space infrastructure to be deployed. The expression is as follow:

$$\mathcal{J} = F(m) \tag{5}$$

where m is the mass of space infrastructure that needs to be deployed by this stakeholder. All numerical experiments performed in this paper are solved through the Python using Gurobi 8.1 solver on an i7-8650U, 1.9GHz platform with 16GB RAM.

For the baseline mission scenario, we assume that the stakeholder plans to perform a space infrastructure deployment mission once a year. There are two equivalent vehicles available at the beginning of each space mission. By default, there are two infrastructure deployment missions in total. Moreover, before the space infrastructure deployment, 10 MT water ISRU system has been deployed in advance on the lunar surface. In this mission scenario, the space mission cost approximation comparison through different regression functions is shown in Fig. 3. We can find that the quadratic function is the best fit for the space mission cost curve with respect to the mass of space infrastructure to be deployed.

Sensitivity analyses are also conducted considering different sizes of ISRU system deployed in advance and a different number of space missions. First, we assume there are 0 MT, 10 MT, and 20 MT ISRU systems deployed in advance. The quadratic mission cost approximation results considering the different size of pre-deployed ISRU systems are shown in Fig. 4. If we fix the ISRU system deployed to 10 MT and change the number of space missions, the quadratic approximation results are shown in Fig. 5. We can find that for both cases, the quadratic function always has a good performance in approximating the mission cost.

In this section, we perform a preliminary analysis to extrapolate an approximation function for space mission cost with respect to the size of space infrastructure deployment demand. The results show that the space mission cost can be approximated as:

$$\mathcal{J} = am^2 + bm + c \quad (6)$$

Starting from the next section, we propose a game theory model for commercial participation in space infrastructure deployment. An analytical incentive design method is developed based on this approximation function.

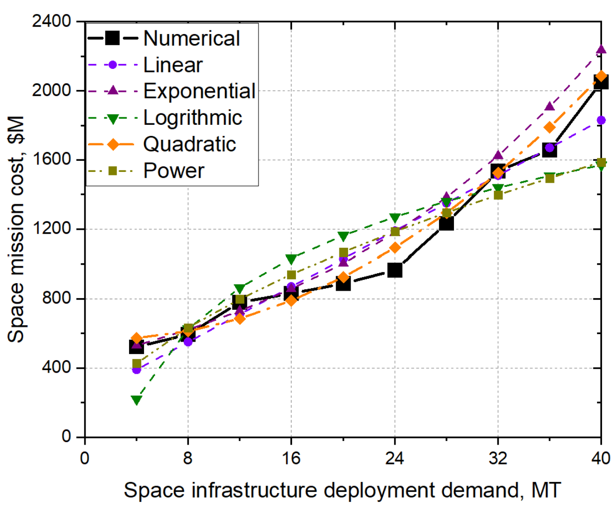

Fig. 3. Space mission cost approximation

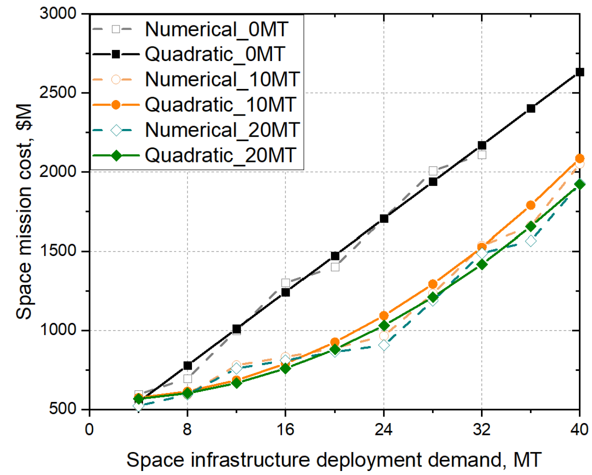

Fig. 4. Quadratic cost approximation for different sizes of ISRU system

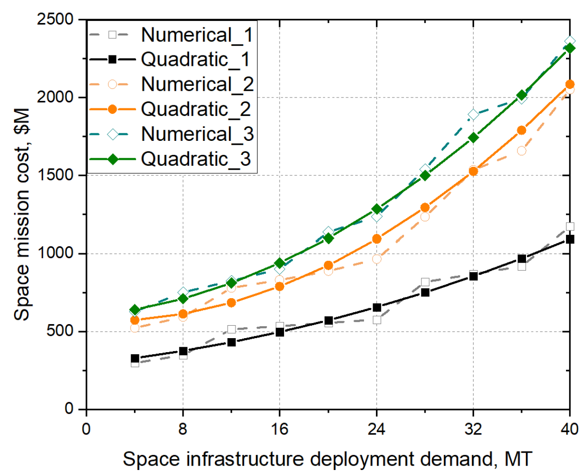

Fig. 5. Quadratic cost approximation for different number of missions

4. Game theory model

In this section, we introduce the problem formulation and game theory model. We consider a specific space logistics problem, where the government or commercial entity as a player that has a mission demand to deploy some space infrastructure or deliver a certain amount of payload to a designated orbit or lunar surface. We assume the total mass of infrastructure or payload to be transported is D. This player can either complete this mission demand by itself only using its own resources or relying on other players (i.e., other active government or commercial entities) to satisfy the mission demand. Other players may have deployed propellant depots, ISRU architectures, and lunar habitats that are able to complete the transportation mission with lower cost. However, their transportation capability is also limited by the transportation vehicle capability and availability because of their own mission objectives. We assume that all players are selfish but rational. Hence, they will only participate in the infrastructure deployment mission when it is beneficial. We define the player as a coordinator/planner who has a specific infrastructure deployment mission demand and would like to leverage other players' capability and resources to complete the space mission.

According to available resources and mission demand, the coordinator can complete the transportation mission partially and deploy the rest of the structure relying on other players. We define a participation coefficient $\alpha \in [0,1]$, denoting the fraction of mission demand to be deployed by other players. Therefore, the transportation mission demand for the coordinator is $(1-\alpha)D$; whereas the mission demand for other players is αD. We assume that the total mission cost to complete the entire infrastructure deployment mission by the coordinator is Q. The baseline mission cost to deliver 1 kg of infrastructure by the coordinator is Q/D. Then, we can define an incentive coefficient $\theta \in [0,1]$. As a rational coordinator, it is only willing to pay the incentive no more than the baseline mission cost. The incentive to deliver 1 kg of infrastructure by other players can be denoted by $\theta Q/D$.

For simplicity, we consider a two-player game with one coordinator and one commercial player. $\mathcal{K}$={coordinator, player} is the set of game players and $[0,1] \times [0,1]$ is the action space for decision variables α and θ. We define $u_o(\alpha,\theta)$ as the utility of the coordinator and $u_p(\alpha,\theta)$ as the utility of the player. The utility is defined as the mission cost savings for the coordinator or the profit for the player. Therefore, the utility of the coordinator can be expressed as

$$u_o(\alpha,\theta) = Q - \mathcal{J}_o(1-\alpha) - \alpha\theta Q \tag{7}$$

And the utility of the player can be written as

$$u_p(\alpha,\theta) = \alpha\theta Q - \mathcal{J}_p(\alpha) \tag{8}$$

where $\mathcal{J}_o$ and $\mathcal{J}_p$ are the space mission cost to complete the designated space infrastructure deployment. Both are functions of only the participation coefficient α because the total infrastructure deployment demand D is known before the optimization. It is a constant in the formulation. By definition, we know $\mathcal{J}_o(1) = Q$. According to the utility functions defined in this section, we propose an analytical incentive design method in the following section.

5. Analytical incentive design methods

In this section, we propose an analytical incentive design method based on the utility functions, Eqs. (1) and (2). The incentive design method is developed according to the Nash bargaining solution as introduced in subsection 5.1. A closed-form solution is identified based on the space mission approximation function. In subsection 5.2, we discuss its properties.

5.1 Nash bargaining solution

The bargaining problem studies how two players share a jointly generated surplus in a non-cooperative game. To formulate the bargaining problem, we need to define the disagreement point, denoted by r, which is a set of strategies that provide the lowest utility as expected by players if the bargaining breaks down. In our problem, the disagreement point is when the commercial player refuses to participate in space infrastructure development and deployment, which means $r = u(\alpha = 0, \theta)$. Then, the Nash bargaining problem can be formulated as

Maximize:

$$\prod_{k\in\mathcal{K}} (u_k(\alpha,\theta) - r_k) \tag{9}$$

Subject to:

$$u_k(\alpha,\theta) - r_k \geq 0 \quad \forall k \in \mathcal{K} \tag{10}$$

$$\alpha, \theta \in [0,1] \tag{11}$$

where Eq. (10) makes sure that the incentive is beneficial to both the coordinator and the player; constraint (11) guarantees that the solution is feasible.

We can solve this nonlinear optimization problem by first applying a logarithm to the objective function and then implementing the Lagrangian multiplier method. We get

$$\mathcal{L}(\alpha,\theta,\lambda,\lambda') = \sum_{k\in\mathcal{K}} ln(u_k(\alpha,\theta) - r_k) - \sum_{k\in\mathcal{K}} \lambda_k\big(r_k - u_k(\alpha,\theta)\big) + \lambda'_1\alpha + \lambda'_2\theta - \lambda'_3(\alpha-1) - \lambda'_4(\theta-1) \tag{12}$$

where $\lambda_k \geq 0, \forall k \in \mathcal{K}$ and $\lambda'_1, \lambda'_2, \lambda'_3, \lambda'_4 \geq 0$. Then, the necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions are

$$\nabla_\alpha \mathcal{L}(\alpha,\theta,\lambda,\lambda') = 0 \tag{13}$$

$$\nabla_\theta \mathcal{L}(\alpha,\theta,\lambda,\lambda') = 0 \tag{14}$$

$$\lambda_k\big(r_k - u_k(\alpha,\theta)\big) = 0 \quad \forall k \in \mathcal{K} \tag{15}$$

$$-\lambda'_1\alpha \leq 0, -\lambda'_2\theta \leq 0, \lambda'_3(\alpha-1) \leq 0, \lambda'_4(\theta-1) \leq 0 \tag{16}$$

where Eqs. (13) and (14) are stationary conditions; Eqs (15) and (16) are complementary slackness conditions.

To identify a close-form solution of the incentive design, according to the preliminary analysis in Sec. 3, we approximate the space mission cost through quadratic functions defined as

$$\mathcal{J}_o(\alpha) = a_1\alpha^2 + a_2\alpha + a_3 \tag{17}$$

$$\mathcal{J}_p(\alpha) = b_1\alpha^2 + b_2\alpha + b_3 \tag{18}$$

From Eq. (15), we know $\lambda_k = 0, \forall k \in \mathcal{K}$. Otherwise, we get $u_k(\alpha,\theta) - r_k = 0$, which means the negotiation breaks down. Equation (16) is satisfied naturally by the definitions of decision variables α and θ. By setting $\lambda'_1, \lambda'_2, \lambda'_3, \lambda'_4 = 0$ and solving Eqs. (13)-(18), we can obtain the Nash bargaining solution

$$\alpha_{NBS} = \begin{cases} \alpha^*, \text{if } \alpha^* \in (0,1] \\ 0, otherwise \end{cases} \tag{19}$$

$$\theta_{NBS} = \frac{Q - \mathcal{J}_o(1-\alpha_{NBS}) + \mathcal{J}_p(\alpha_{NBS})}{2\alpha_{NBS}Q} \tag{20}$$

where $\alpha^* = \frac{2a_1+a_2-b_2}{2a_1+2b_1}$. It is a value calculated based on the coefficients in space mission cost approximation functions. By definition, we know $\alpha_{NBS} \in [0,1]$. Hence, $\theta_{NBS} \in [0,1]$ is always true in Eq. (20). Therefore, Eqs. (19) and (20) always a feasible strategy.

5.2 Incentive design properties

For the closed-form expression of the incentive design model, we can identify two properties as follows.

Theorem 1. *The Nash bargaining solution ($\alpha_{NBS}, \theta_{NBS}$) maximizes the total social welfare, which is the total utility of the coordinator and the player.*

$$\sum_{k\in\mathcal{K}} u_k(\alpha_{NBS}, \theta_{NBS}) \geq \sum_{k\in\mathcal{K}} u_k(\alpha', \theta') \quad \forall \alpha', \theta' \in [0,1] \tag{21}$$

PROOF. The optimization problem to maximize the total utility of all players in the system can be formulated as follow:

Maximize:

$$\sum_{k\in\mathcal{K}} u_k(\alpha, \theta) \tag{22}$$

Subject to:

$$u_k(\alpha, \theta) \geq 0 \quad \forall k \in \mathcal{K} \tag{23}$$

$$\alpha, \theta \in [0,1] \tag{24}$$

After implementing the Lagrangian multiplier method, we get

$$\mathcal{L}(\alpha, \theta, \lambda, \lambda') = \sum_{k\in\mathcal{K}} u_k(\alpha, \theta) + \sum_{k\in\mathcal{K}} \lambda_k u_k(\alpha, \theta) + \lambda_1' \alpha + \lambda_2' \theta - \lambda_3'(\alpha - 1) - \lambda_4'(\theta - 1) \tag{25}$$

where $\lambda_k \geq 0, \forall k \in \mathcal{K}$ and $\lambda_1', \lambda_2', \lambda_3', \lambda_4' \geq 0$. The necessary and sufficient KKT conditions are the same as Eqs. (13)-(16) when setting $r_k = 0, \forall k \in \mathcal{K}$. In this scenario, θ can be any value because it is eliminated in the objective function. By solving these equations, we obtain the optimal α^* that maximizes the system total utility:

$$\alpha_{social}^* = \begin{cases} \alpha^*, \text{if } \alpha^* \in [0,1] \\ 0, otherwise \end{cases}$$

where $\alpha^* = \frac{2a_1+a_2-b_2}{2a_1+2b_1}$. We achieve the same solution as the Nash bargaining problem,

$$\alpha_{social}^* = \alpha_{NBS}$$

Therefore, we can claim that the Nash bargaining solution of the incentive design also maximizes social welfare. ■

Besides the total utility obtained by the system, we also care about the fairness of the benefit distribution. The incentive can be designed focusing on the fairness leveraging the maximin strategy. The purpose of the maximin strategy is to optimize the utility distribution among players. It achieves this goal by maximizing the minimum utility obtained by each player. For the Nash bargaining solution shown in Eqs. (19) and (20), we can also prove the following theorem.

Theorem 2. *The Nash bargaining solution ($\alpha_{NBS}, \theta_{NBS}$) maximizes the minimum utility that is received by each player.*

$$\min\{u_k(\alpha_{NBS}, \theta_{NBS}), \forall k \in \mathcal{K}\} \geq \min\{u_k(\alpha', \theta'), \forall k \in \mathcal{K}\} \quad \forall \alpha', \theta' \in [0,1] \tag{26}$$

PROOF. The optimization problem to maximize the minimum utility received by each player can be formulated as:

Maximize:

$$\min\{u_k(\alpha, \theta), \forall k \in \mathcal{K}\} \tag{27}$$

Subject to:

$$u_k(\alpha, \theta) \geq 0 \quad \forall k \in \mathcal{K} \tag{28}$$

$$\alpha, \theta \in [0,1] \tag{29}$$

We can define a variable $Z = \min\{u_k(\alpha, \theta), \forall k \in \mathcal{K}\}$. Then, we know

$$Z \leq u_k(\alpha, \theta) \quad \forall k \in \mathcal{K}$$

The maximin strategy formulation can be rewritten as,

Maximize:

$$Z \tag{30}$$

Subject to:

$$Z \leq u_k(\alpha, \theta) \quad \forall k \in \mathcal{K} \tag{31}$$

$$u_k(\alpha, \theta) \geq 0 \quad \forall k \in \mathcal{K} \tag{32}$$

$$\alpha, \theta \in [0,1] \tag{33}$$

Then, the Lagrangian of this formulation can be written as

$$\mathcal{L}(\alpha, \theta, \eta, \lambda, \lambda') = Z - \sum_{k\in\mathcal{K}} \eta_k \big(Z - u_k(\alpha, \theta)\big) + \sum_{k\in\mathcal{K}} \lambda_k u_k(\alpha, \theta) + \lambda_1' \alpha + \lambda_2' \theta - \lambda_3'(\alpha - 1) - \lambda_4'(\theta - 1) \tag{34}$$

The necessary and sufficient KKT conditions are,

$$\nabla_\alpha \mathcal{L}(\alpha, \theta, \eta, \lambda, \lambda') = 0 \tag{35}$$

$$\nabla_\theta \mathcal{L}(\alpha, \theta, \eta, \lambda, \lambda') = 0 \tag{36}$$

$$\eta_k\big(Z - (u_k(\alpha, \theta) - r_k)\big) = 0 \quad \forall k \in \mathcal{K} \tag{37}$$

$$\lambda_k\big(r_k - u_k(\alpha, \theta)\big) = 0 \quad \forall k \in \mathcal{K} \tag{38}$$

$$-\lambda_1' \alpha \leq 0, -\lambda_2' \theta \leq 0, \lambda_3'(\alpha - 1) \leq 0, \lambda_4'(\theta - 1) \leq 0 \tag{39}$$

Solving these equations, we obtain the same solution as the Nash bargaining solution.

$$\alpha_{fair} = \alpha_{NBS}$$

$$\theta_{fair} = \theta_{NBS}$$

■

In this section, we propose and identify a closed-form expression of an incentive design method based on the Nash bargaining solution. We show that the proposed incentive design method also maximizes the total utility of the system and the minimum utility received by each player. In the next section, we evaluate the performance of the proposed incentive design method and demonstrate its properties through a lunar exploration case study.

6. Lunar exploration case study

In this section, we perform a case study on lunar exploration campaign to demonstrate the performance of the proposed incentive design method. First, in subsection 6.1, we introduce the numerical formulation to maximize the system total utility and the minimum utility received by each player. Then, we define the problem scenario for analysis in subsection 6.2. Finally, in subsection 6.3, we run simulations to compare and discuss the impact of the proposed incentive design method.

6.1 Numerical formulation

Leveraging the same network-based space logistics optimization model as discussed in Sec. 2, we can develop numerical-based incentive design methods to maximize social welfare and fairness.

In this game, there are two players considered, $\mathcal{K}$={coordinator, player} (index: k). Therefore, we need to define a different transportation system for each of them, which leads an additional index for commodity flow variable $\boldsymbol{x}_{kvijt}$, the cost coefficient $\boldsymbol{c}_{kvijt}$, and the mission demand vector $\boldsymbol{d}_{kit}$. The problem formulation for social welfare maximization is shown as follows.

Maximize:

$$\sum_{k\in\mathcal{K}} u_k(\alpha,\theta) = Q - \sum_{k\in\mathcal{K}} \mathcal{J}_k \qquad (40)$$

Subject to:

$$\mathcal{J}_k = \sum_{(v,i,j,t)\in\mathcal{A}} \boldsymbol{c}_{kvijt}{}^T \boldsymbol{x}_{kvijt} \quad \forall k\in\mathcal{K} \qquad (41)$$

$$\sum_{(v,j):(v,i,j,t)\in\mathcal{A}} \boldsymbol{x}_{kvijt} - \sum_{(v,j):(v,j,i,t)\in\mathcal{A}} G_{kvji}\boldsymbol{x}_{kvji(t-\Delta t_{ji})} \le \boldsymbol{d}_{kit} + \boldsymbol{d}'_{kit}(\alpha)$$

$$\forall i\in\mathcal{N}\ \ \forall t\in\mathcal{T}\ \ \forall k\in\mathcal{K} \quad (42)$$

$$H_{kvij}\boldsymbol{x}_{kvijt} \le \boldsymbol{0}_{l\times 1} \quad \forall (v,i,j,t)\in\mathcal{A}\ \ \forall k\in\mathcal{K} \qquad (43)$$

$$\begin{cases} \boldsymbol{x}_{kvijt} \ge \boldsymbol{0}_{p\times 1} & \text{if } t\in W_{kij} \\ \boldsymbol{x}_{kvijt} = \boldsymbol{0}_{p\times 1} & \text{otherwise} \end{cases}$$

$$\forall (v,i,j,t)\in\mathcal{A}\ \ \forall k\in\mathcal{K} \qquad (44)$$

$$\boldsymbol{x}_{kvijt} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}_{kvijt}, x_n \in \mathbb{Z}_+ \ or\ \mathbb{R}_+ \quad \forall n\in\{1,\dots,p\}$$

$$\forall (v,i,j,t)\in\mathcal{A}\ \ \forall k\in\mathcal{K}$$

This is a formulation developed based on the traditional space logistics optimization model. The main difference comes from the mission demand. The demand vector $\boldsymbol{d}_{kit}$ denotes the original mission demand depending on each player's mission goal. Another demand vector $\boldsymbol{d}'_{kit}(\alpha)$ denotes additional infrastructure deployment demand determined based on the participation coefficient α.

Similarly, by changing the objective function in the formulation Eqs. (40)-(44), we obtain the maximin formulation focusing on fairness.

Maximize:

$$\underset{k\in\mathcal{K}}{\text{Min}}\{u_k(\alpha,\theta)\} \qquad (45)$$

Subject to:

Eqs. (41)-(44)

6.2 Problem setting

Assuming a cislunar exploration campaign with two potential players. Player 0, which is the coordinator, has annually infrastructure deployment demands; whereas player 1 focuses on its own lunar exploration mission. For simplicity, we assume player 1 is the only player who has the ability to develop and deploy ISRU systems on the lunar surface to support space transportation. Player 1 is able to participate in the infrastructure deployment mission if player 0 provides enough incentives. The cis-lunar transportation network, the mission operation assumptions, and the mission cost model are the same as the preliminary analysis in Sec. 3, as shown in Fig. 2, Table 1, and Table 2.

At the beginning of the space mission, we assume that player 1 has already built a lunar base with 5 MT water ISRU system. For the baseline mission scenario, player 0 plans to deploy 40 MT lunar habitat and infrastructure to the Moon every year. The baseline mission demands and supplies for this case study are shown in Table 3. Note that, none of the players can complete this space infrastructure deployment mission independently under current mission scenario using available resources. Therefore, we manually define the baseline mission cost of player 0, which is the coordinator, as $3,000M. This is a dummy value for utility calculation and does not impact the result of decision variables α and θ.

6.3 Numerical experiments

Before implementing the incentive design model, we need to identify the quadratic space mission cost approximation function. For the baseline mission scenario, we assume there are two infrastructure deployment missions and both players have two LH2/LOX vehicles available at the beginning of each space mission. The mission cost approximation results are shown in Fig. 6. The space mission cost approximation function can be expressed as

$$\mathcal{J}_o(\alpha) = 0.0005\alpha^2 + 2306.3\alpha + 313.75 \qquad (46)$$

$$\mathcal{J}_p(\alpha) = 1214.2\alpha^2 + 1021.6\alpha + 398.54 \qquad (47)$$

Table 3 Mission demands and supplies

Actors	Payload Type	Node	Time, day	Demand, MT
Player 0	Infrastructure	Earth	357 (repeat annually)	+40
	Infrastructure	Moon	360 (repeat annually)	-40
	Propellant	Earth	All the time	+∞
Player 1	ISRU plant, maintenance, and Propellant	Earth	All the time	+∞
	ISRU plant	Moon	0	+5

Table 4 Incentive design result comparison

Methods	α^*	θ^*	Total utility, \$M	Player 0 utility, \$M	Player 1 utility, \$M
Analytical, NBS	0.529	0.919	336.04	168.02	168.02
Numerical, social welfare	0.534	——	337.21	——	——
Numerical, fairness	0.534	0.914	337.21	168.61	168.61
Numerical, single stakeholder	——	——	651.40	——	——

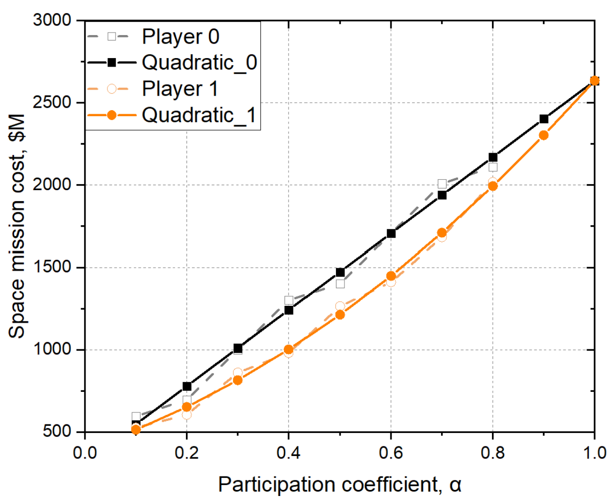

Fig. 6. Space mission cost approximation

Then, we substitute the approximation functions to the incentive design model, Eqs. (19) and (20), we can get the analytical incentive design solution. The result comparison between the analytical incentive design solution and the results from numerical formulations are shown in Table 4. First, we can find that the analytical incentive design solution is very close to the numerical results obtained through maximizing the total social welfare and fairness, respectively. It demonstrates the properties we propose in Sec. 5. Moreover, we can obtain the analytical incentive design solutions instantaneously after extrapolating space mission cost approximation functions. To obtain the data points for space mission cost approximation, we only need to solve the mission planning problem for each player each mission independently. We do not need to solve the space logistics optimization problem considering all players and missions all-at-once. It significantly improves the computational efficiency by avoiding the curse of dimensionality in the optimization.

Furthermore, the single stakeholder transportation system, which contains only one player who has a top-down control of all resources, shows a much better performance than those transportation systems considering multiple players. To analyze the impact of multi-player decision-making environment, we perform a sensitivity analysis with respect to different sizes of ISRU system deployed in advance before infrastructure deployment missions. In this case, we assume that more ISRU system may be deployed to the Moon to support the space mission. However, the new ISRU system manufacturing and transportation cost will be deducted from the final utility. The total utility and the mass of newly deployed ISRU system are compared between a single-player system and a two-player system with one coordinator and one commercial player. Note that, for the two-player system, only the commercial player has the ability to operate and deploy new ISRU systems. The comparison results are shown in Fig. 7. We can find that as the increase of the pre-deployed ISRU system size, the total utility increases and the mass of newly deployed ISRU system decreases. Moreover, the total utility of the single-player system is always higher than the two-player transportation system. The reason is that in the two-player transportation system, the commercial player is the only entity to undertake ISRU transportation and deployment cost. It will deploy more ISRU system only when more infrastructure deployment demands are offered and a higher incentive is provided. When the mass of pre-deployed ISRU system is relatively high (i.e., above 3 MT as shown in Fig. 7) such that the commercial player is able to deploy the entire infrastructure system to the Moon, the coordinator can always increase its own utility by offering higher incentive to the commercial player to increase its participation coefficient. Because in this scenario, the commercial player can always complete the space infrastructure deployment with a lower mission cost.

Therefore, eventually, the commercial player deploys the entire infrastructure and more ISRU system is deployed during the mission than in the single-player transportation system. The total utility difference between the single-player system and the two-player system is also reduced significantly.

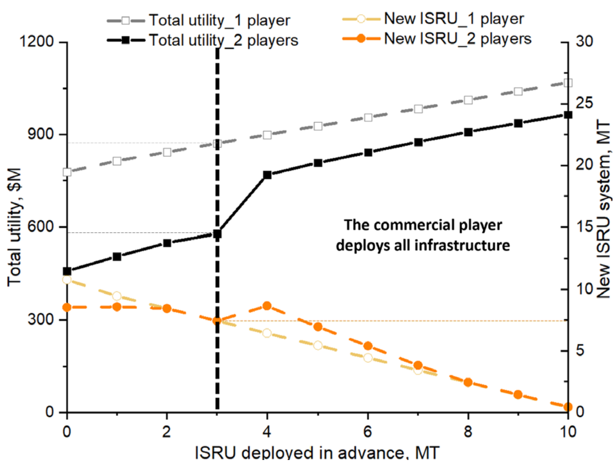

Fig. 7. The impact of the multi-player decision-making environment

To evaluate the impact of the ISRU system on the incentive design decisions, we perform a sensitivity analysis on the participation coefficient α with respect to the total infrastructure deployment demand and the pre-deployed ISRU system size. The result is shown in Fig. 8. We can find that when the space infrastructure deployment demand is low, the commercial player deploys the entire infrastructure leveraging its ISRU system. When the space infrastructure size is larger than the commercial player's deployment capacity, the coordinator and the player both participate in the infrastructure deployment mission and deploy a fraction of the infrastructure, respectively. Furthermore, more ISRU system deployed in advance provides the commercial player better support in space transportation, which leads to a higher participation coefficient. However, the maximum infrastructure deployment capacities of the coordinator and the player are the same because we assume that they both have two equivalent vehicles at the beginning of each mission. Therefore, the participation coefficient α converges to 0.5 eventually as the increase of the infrastructure deployment demand.

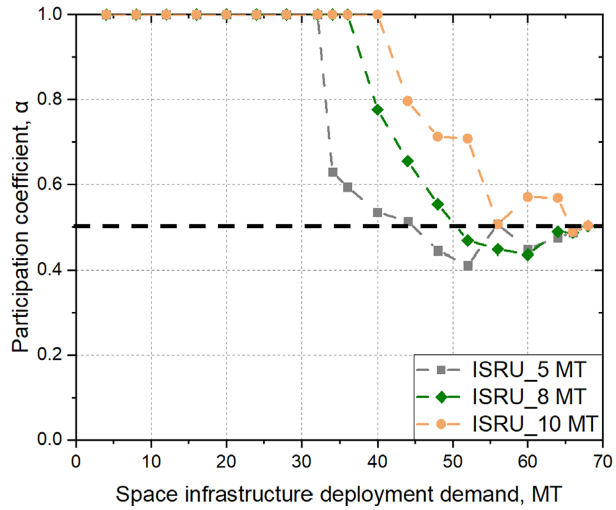

Fig. 8. Participation coefficient sensitivity analysis

7. Conclusions

To stimulate commercial participation in space logistics infrastructure development and deployment, this paper proposes an analytical incentive design method based on the Nash bargaining solution. We prove its properties that maximize social welfare and fairness at the same time while minimizing the incentive. A lunar exploration case study is performed to demonstrate its properties and compare its performance with numerical incentive design methods. Results show that this analytical incentive design method can provide accurate decisions instantaneously after extrapolating the mission cost approximation model. Moreover, sensitivity analysis shows that single-player transportation systems are always more efficient than the two-player transportation systems because, in a single-player system, the whole system will undertake the ISRU deployment cost instead of one of the players. However, when more ISRU system is deployed in advance or the ISRU system has higher productivity, the utility difference between single-player and two-player transportation systems can be reduced significantly. The commercial player is also able to achieve a higher participate coefficient and higher utility.

Future research includes the consideration of uncertainties in the incentive design, including demand changes, spacecraft flight or rocket launch delay, and infrastructure performance uncertainties. Further implementations can also be explored to consider multiple commercial players with different technology strengths.

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